THE CHINESE EUCLID AND ITS EUROPEAN CONTEXT

Peter ENGELFRIET

It is tempting to regard the introduction of Euclidian geometry into China as a confrontation between Chinese and Western thought. It is clear that Euclid has had an immeasurable influence on science and philosophy in the West, and even on culture in general. It served as the most perfect example of what rational thinking may achieve, and it became a canonical model of method.

On the other hand, if we accept Euclid as one of the pillars of Western scientific thought for having introduced the concept of rigorous proof and the deductive method, we seem to have isolated an important element in the "test-tube" of comparative history — an element that could help to explain the different ways science developed in these two parts of the world. In this perspective, the translation of Euclid into Chinese in 1607 by Matteo Ricci and Xu Guangqi,[1] could, in principle, qualify as a test-case for defining some basic differences between Chinese and Western thought. Failures to convey the spirit of Euclid into classical Chinese could reveal any linguistic incompatibilities, and Chinese reactions could bring to light different patterns of thought.

However, before setting up the "experiment", we should have to account for some bold assumptions. In the first place, we would have to accept Euclid as an integral part of Western thought. Yet, for example, the first Euclidian material to appear in English, in 1551, was preceded by the following warning:

> For nother is there anie matter more straunge in the englishe tungue, then this whereof never booke was written before now, in that tungue, and therefore oughte to delite all them, that desire to understand straunge matters, as most men commonlie do.[2]

In the second place, we would have to establish that the translation by Ricci was the best possible representation of Euclid. Now, Jean-Claude Martzloff, whose ground-breaking work on the introduction of Western mathematics into China has demonstrated the

1. Matteo Ricci and Xu Guangqi, *Jihe yuanben*, Peking, 1607. All references are to the reprint in the *Wenyuan ge* edition (Taibei, 1983) of the *Siku quanshu*, vol. 798, pp. 563-932.

2. Robert Recorde, *The Pathway to Knowledge, Preface*, London, 1551; photocopy-reprint, Amsterdam, 1974. Cited in J.E.Murdoch, "Euclid: Transmission of the *Elements*", in C. Gillispie ed., *Dictionary of Scientific Biography*, vol. IV, 1971, pp. 437-459, p. 449.

importance of detailed studies in this field, has already drawn attention to the fact that it was not Euclid who was translated but Clavius.[3]

In what follows, I want to elaborate on these two themes. The first one requires a more general approach. I will not be able to present more than a sketch of the place and nature of mathematics in sixteenth and early seventeenth century Europe. I shall briefly consider mathematical practice and questions of epistemology and method, indicating the potential relevance of these considerations for the translation. They also throw some light on the Jesuit flirtation with science and mathematics. Next, I shall give an impression of the very complicated and tortuous integration of Euclid in Western European thought. This will serve as a prelude to a closer look at the Ricci-Xu translation. I have taken an example from the translation, to show that the true "spirit" of Euclid has not always been conveyed. In doing so, I shall draw attention to some problematic aspects of the Greek approach to mathematics.

First of all, it should never be forgotten that Euclid was a product of classical Greek culture, written around 300 B.C. After leaving practically no marks on Roman culture, it was only in the twelfth and thirteenth centuries that the Greek texts became available to Western Europe in translations that were largely made from the Arabic versions. At that time they posed fundamental problems of comprehension and assimilation. According to Mahoney, only in the latter part of the fifteenth and the sixteenth centuries did "European mathematicians start thinking in Greek mathematical terms again", and then only partially.[4] Not only is the textual transmission of Euclid extremely complex,[5] but also interpretations of the work varied widely; interpretations concerning the importance and nature of mathematics, the ontological status of mathematical objects, requirements of proof, the nature of proof etc., and also in a more practical way, on items such as its usefulness and its pedagogical value.

In speculating on the reason why in China only the first six books were translated, we might be tempted to conclude that the Jesuits were only interested in spreading religion and that the first six books would suffice for that purpose. But once we realize that six-book-versions were more the rule than the exception in Europe, we are forced to reassess our judgement. In fact, toward the 1570s someone versed in the first six Books of Euclid might well be considered an expert, although this situation was rapidly changing.[6]

Let me refer, again, to the work of Jean-Claude Martzloff. In his excellent study on the mathematical works of Mei Wending he has shown the influence of Euclidian geometry

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3. J.-C. Martzloff, "Matteo Ricci's Mathematical Works and Their Influence", in Lo Kuang ed., *Collected Essays of the International Symposium on Chinese-Western Cultural Interchange in Commemoration of the 400th Anniversary of the Arrival of Matteo Ricci S.J. in China*, Taipei, Fujen University, 1983, pp. 437-453, esp. pp. 443-444.

4. M.S. Mahoney, "Mathematics", D. Lindberg ed., *Science in the Middle Ages*, Chicago, 1978, pp. 145-178, p. 146.

5. Murdoch, *op.cit.*, p. 437: "No other work scientific, philosophical, or literary has, in making its way from antiquity to the present, fallen under an editor's pen with anything like an equal frequency."

6. M.S. Mahoney, *The Mathematical Career of Pierre de Fermat*, Princeton, 1973, pp. 12-13: "As had been the case in the high Middle Ages, university students of the sixteenth and early seventeenth centuries learned little more than the first six Books of Euclid's Elements as preparation for reading the introductory sections of Ptolemy's Almagest."

on the most famous mathematician of the Qing.[7] In another article he has evaluated in a more general way the Chinese understanding of Euclidian methodology.[8] His work leaves no doubt that that understanding was very limited, and that there was a tendency to ignore the deductive machinery and to rearrange the material. But we should also take into consideration the many European "Euclid-made-simpler" editions. For example, the famous Parisian pedagogue and logician Pierre de la Ramée (1515-1572), who devoted his life to the restoration of the seven liberal arts, published radically different presentations of Euclidian geometry.[9] In his *Scholae Mathematicae*[10] he severely criticized Euclid, and in his *Geometria*[11] he rearranged the material, imposing a completely different ordering by grouping it in chapters on magnitude, line, angle, figure, etc.. By doing this he completely abandoned the deductive structure of Euclid, showing himself to be blind to the dependence of later theorems on previous theorems and principles. He left out most of the proofs, and changed the meaning and function of the terms "axiom", "postulate" and "theorem". In fact, he only started studying the proofs of Euclid sixteen years after his edition of the *Elements*. On the other hand it cannot be said that he moved on the periphery of the mathematical world. Verdonk mentions three reeditions (the last one in 1627), six abstracts, and three translations, apart from the influence he had on contemporary mathematicians.[12] But the most remarkable feature is the observation by Verdonk that Ramus, though several times attacked by his contemporaries for his ideas on method, was never accused of a faulty understanding of the mathematical mode of thought.[13] In fact, his *Scholae* were the result of intensive communication with other scholars.

Another interesting aspect of Ramus is his insistence on the importance of practice. He was very proud of his contacts with craftsmen and artists, and even visited the working-places in the rue Saint-Denis, noting which theorems were actually being used in practice. He criticized Plato for having made a radical separation between theory and practice.[14] Of course, in the sixteenth century, the practical side of mathematics was very much in the ascendence.

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7. J.-C. Martzloff, *Recherches sur l'oeuvre mathématique de Mei Wending (1633-1721)*, Paris, 1981. In the fifth chapter he discusses Mei's reactions to Euclidian material and how he tried to reinterpret Euclid in the context of Chinese mathematics.

8. J.-C. Martzloff, "La compréhension chinoise des méthodes démonstratives euclidiennes au cours du XVII^{e} siècle et au début du XVIII^{e}", in *Actes du II^{e} Colloque International de Sinologie: les rapports entre la Chine et l'Europe au temps des Lumières, Chantilly, sept. 1977*, Paris, 1980, pp. 125-143.

9. For the mathematical work of Pierre de la Ramée I rely myself on J.J. Verdonk, *Petrus Ramus en de Wiskunde (Petrus Ramus and Mathematics)*, Assen, 1966. Verdonk gives a very detailed analysis of the mathematical work of de la Ramée. His study is invaluable for its information on sixteenth century background.

10. *P. Rami Scholarum Mathematicarum, libri unus et triginta*, Basel, 1569.

11. *P. Rami Arithmeticae libri duo: Geometriae septem et viginti*, Basel, 1569.

12. Verdonk, *op. cit.*, pp. 226-231.

13. *Ibidem*, p. 376.

14. Compare R. Hooykaas, *Humanisme, science et réforme. Pierre de la Ramée (1515-1572)*, Leiden, 1958, pp. 58-59: "D'après Ramus la vraie splendeur de la géométrie ne luit pas dans les préceptes et les théorèmes, mais dans l'emploi qu'en font les peintres, les mécaniciens, etc.; sa fin ultime et suprême est de 'bien mesurer' et, par conséquent, les éléments de la géométrie ne sont posés ou enseignés que dans ce but."

This European scene of bustling technological activity is echoed in Ricci's preface to the *Jihe yuanben*.[15] He exalts at length the tremendous usefulness of mathematics for almost every aspect of human life, from digging wells and arranging a battle-order to the curing of patients. Was he just "selling" mathematics? Actually, from a European point of view, this preface is highly conventional. A eulogy on the great benefits of mathematics for society may be found in many prefaces to mathematical works in the sixteenth century. It is a central theme in what has been labelled "the Renaissance of Mathematics",[16] and undoubtedly the seeking of patronage was a strong motive for stressing such utility. It is, indeed, a far cry from the Platonic scorn for the practical side of mathematics. The "co-operation of head and hand"[17] that was beginning to take form is very conspicuous in Christophorus Clavius' Latin translation of and commentary on Euclid, the work Ricci and Xu used as the basis for their translation.[18] Clavius is probably best-known for the role he played in the Gregorian reform of the Calendar. He was one of the three astronomical experts in the congregation of nine that was established in 1580 by Pope Gregory XIII to construct the new calendar.[19] He thus set the stage for an intense Jesuit involvement in astronomy. His example made clear that mathematics was a road to power that had to be reckoned with. But Euclid is not directly a work on astronomy, and it certainly meant more to Clavius than worldly benefits. So how can we explain that a Christian missionary, whose main goal was to spread the Gospel and bring the light of Christianity to the heathen, decided to give such extraordinary prominence to Euclid? Reflection on the reasons for this belief in the transforming power of mathematics may provide an occasion to give an impression of the place of Euclid in sixteenth and early seventeenth century Europe.

These reasons coincide with distinctly different aspects of Euclid that are important to distinguish.

In the first place, the *Elements* constitute a collection of basic mathematical facts in the form of theorems. As Clavius explains in his *Prolegomena*, they are as the letters of the alphabet one has to master before being able to write. They formed the basis for more advanced mathematical studies or for applied mathematics, that could be taken for granted without needing further explication. In this sense they were a sourcebook, or *yuanben*. As Ricci writes in the preface: "At my university, all the books of the many different branches of mathematics take this book as their starting-point. Every principle, every theory that is

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15. This preface has not been included in the *Siku quanshu*. It can be found in *Tianxue chuhan*, vol. 4, p. 1921ff, Taipei, 1965 (Reprint; first edition 1629). The preface has been translated by P. D'Elia, "Presentazione della prima traduzione Cinese di Euclide", *Monumenta Serica* XV (1956), pp. 161-202.

16. P.L. Rose, *The Italian Renaissance of Mathematics: Studies on Humanists and Mathematicians from Petrarch to Galileo*, Geneva, 1975.

17. I took the expression from R. Hooykaas, *Religion and the Rise of Modern Science*, Edinburgh, 1972, p. 88.

18. Christophorus Clavius, *Euclidis Elementorum libri XV Accessit XVI de Solidorum Regularium cuiuslibet intra quodlibet comparatione, Omnes perspicuis Demonstrationibus, accuratisque Scholiis illustrati, ac multarum rerum accessione locupletati*, Cologne, 1591, third edition. Unfortunately, I have not been able to consult the first edition (Rome, 1574). I did consult the second edition (Rome, 1589), but did not find differences with the third. The major differences with the first edition are mentioned by Clavius in the second edition. There are strong reasons to believe that Ricci used the first edition. See note 78.

19. See U. Baldini, "Christoph Clavius and the Scientific Scene in Rome", in G.V. Coyne, S.J., M.A. Hoskin and O. Pedersen ed., *Gregorian Reform of the Calendar. Proceedings of the Vatican Conference to Commemorate its 400th Anniversary 1582-1982*, Vatican City, 1983, pp. 137-169, spec. pp. 137-138.

being developed, cites this book as its proof... It suffices to cite 'Book *x*, theorem *y*'."[20] Ricci wanted to lay a solid foundation for his whole project of introducing science. One of his main aims was to get in charge of the calendar-reform by showing the superiority of Western methods, that leaned heavily on geometrical theorems. In his time Greek geometry was still considered as the basis of astronomy.

The second reason is of a more metaphysical nature and concerns the question that has so much occupied Western philosophers: what is mathematics about? What is the nature of mathematical objects and how do they relate to physical objects? Clavius, in his *Prolegomena*, accords to mathematics, as far as its subject-matter is concerned, a place intermediate between physics (the study of nature) and metaphysics. The mathematical sciences are concerned with things that, while immersed in matter, are considered apart from matter, whereas metaphysics is both *qua* object (*res*) and *qua* method (*ratio*) completely separated from sensible matter, and physics vice versa. In this view mathematical studies become a "stepping-stone" to metaphysics and theology:

> The mathematical disciplines have to be considered not only as useful, but also as fundamental to the mastery of the other arts, and to the proper ordering and administration of the State. In fact, as Proclus has shown with elegance, nobody can accede to metaphysics if not by way of mathematics. For if we try, without any intermediary, to elevate the powers and sharpness of our intellect from sensible things that the physicist considers, to things apart and separated from all sensible matter that the metaphysician considers, we will be blinded, like one who is thrown out from an obscure prison, where he has been for a long time, into the full light of the sun.[21]

The Platonic influence here is unmistakable. Mathematical objects are one step up on the "chain of being"[22] to the pure light of God. By showing the more visible objects of mathematics, Ricci wanted to prepare the minds of the Chinese for the acceptance of the higher truths of Christianity.[23]

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20. Ricci, *op. cit.*, pp. 1936-1937.

21. Clavius, *op. cit.*, *Prolegomena* (unnumbered): "Non solùm utiles, verùm etiam necessariae admodum censeri debent disciplinae Mathematicae, cùm ad alias artes perfectè perdiscendas, tum ad rem etiam publicam rectè instituendam, et administrandam. Neque enim ad Metaphysicam, ut eleganter ostendit Proclus, ulli patet aditus, nisi per Mathematicas disciplinas. Nam si ê rebus sensibilibus, quas Physicus considerat, ad res ab omni materia sensibili secretas, seiunctasque, quas contemplatur metaphysicus, vires, aciemque nostri intellectus attollere absque ullo medio tentemus; nosmetipsos excaecabimus, non secus, ac ei contingit, qui è carcere aliquo tenebricoso, in quo diu latuit, in lucem Solis clarissimam emittitur." For further discussions of the content of the *Prolegomena* see the following references: F.A. Homann, S.J., "Christopher Clavius and the Renaissance of Euclidean Geometry", *Archivum Historicum Societatis Iesu*, 52 (1983), pp. 233-246; W.A. Wallace, *Galileo and his Sources. The Heritage of the Collegio Romano in Galileo's Science*, Princeton, N.J., 1984, Chapter 3; P. Dear, "Jesuit Mathematical Science and the Reconstitution of Experience in the Early Seventeenth Century", *Studies in the History and Philosophy of Science*, XVIII-2 (1987), pp. 133-175, spec. pp. 136-141.

22. Cf. A.O. Lovejoy, *The Great Chain of Being*, Cambridge, Mass., 1936.

23. Cf. I. Dunyn-Szpot, *Historiae Sinarum*, II, 2, n°VI; ms. *circa* 1700, *Japonica-Sinica* 102 (Archivum Romanum Societatis Iesu).

This Platonic ring may come as a surprise considering the Aristotelian signature of the Jesuits, but it only shows the syncretism of all sixteenth century philosophy.[24] Research has increasingly made clear how unsatisfactory any attempt at dividing this period up into different schools must be.[25]

Proclus (410-485), whom Clavius is citing here, was once head of the Neoplatonic Academy of Athens, and he wrote many commentaries on the works of Plato. He also wrote a very influential commentary on the first Book of the *Elements*. This commentary had only recently become available as an appendix to the 1533 *editio princeps* of the Greek text of Euclid by Grynaeus in Bale — but above all through the Latin translation by Francesco Barozzi in Venice, published in 1560.[26] Apart from the many *loci mathematici* in Aristotle and Plato, Proclus was almost the only source for a wider interpretation and understanding of Euclid. Clavius' *Prolegomena*, apart from citing Proclus (and *Divinus Plato*) very frequently, was deeply influenced by Proclus in its attempt to give mathematics a major place in the scheme of knowledge.[27]

Clavius was certainly not the only Jesuit to show an adoption of Platonic ideas. Antonio Possevino for example, one of the major figures in the formation of the *Ratio Studiorum*, wrote in his *Bibliotheca selecta qua agitur de ratione studiorum* (1593):

> For in the *Timaeus* Plato makes God construct the soul of the world from arithmetical ratios and proportions and its body from geometrical shapes. [...] And these things would certainly seem more than enough to excite minds towards those disciplines, were it not that two other things add to their reputation: the one said by Plato, which (so Plutarch says) smacks of Plato's character, although it does not survive in his dialogues, namely that 'God above all geometrizes'; the other having regard to their origins, for they have spread down from the most ancient patriarch Abraham to other men. Indeed He, by whose divine mind everything is providently administered, for the safety and presentation of all, has been said by Plato to govern and control this universe by geometrical proportion.[28]

Indeed, this is nothing else than the theme of the Book of Nature, written in mathematical characters, that was such an inspiration to many of the great scientists of the next century. It is the background to Clavius' proof in his commentary on the *Sphaera* of

24. On Jesuits and Platonism see A.C. Crombie, "Mathematics and Platonism in the Sixteenth Century Italian Universities and in Jesuit Educational Policy", in Y. Maeyama and W.G. Saltzer ed., *Prismata. Festschrift für Willy Hartner*, Wiesbaden, 1977, pp. 63-94.

25. On this see especially C.B. Schmitt, *Aristotle and the Renaissance*, Cambridge, Mass., 1983. He, for example, prefers to speak about "Aristotelianisms" in the plural (p. 10).

26. For an English translation see Proclus, *A commentary on the first Book of Euclid's Elements*. Transl. with introduction and notes by Glenn R. Morrow, Princeton, N.J., 1970.

27. Cf. F.A. Homann, *op. cit.* p. 238.

28. A. Possevino, *Bibliotheca selecta qua agitur de ratione studiorum in historia, in disciplinis, in salute omnium procuranda*, 2 Partes, Romae, 1593, Book XV, *De Mathematicis*. The translation is Crombie's, in A.C. Crombie, *op. cit.*, p. 71.

Sacrobosco[29] — also partly translated by Ricci into Chinese[30] — that, of all possible solids with equal surface the sphere contains the greatest volume, an attempt to prove mathematically the perfection of God's creation of the spherical Heavens. Also, in the famous Jesuit commentaries of Coimbra on the works of Aristotle, we find, among the definitions of God, that "God is an infinite sphere whose center is everywhere and circumference nowhere".[31] This definition was drawn from a twelfth century pseudo-Hermetic work. According to Schmitt, the "invasion of hermetic material into Aristotelian contexts is frequent throughout the Renaissance" and the related doctrine of the *prisca sapientia* was deeply "woven into the fabric of the sixteenth century". Of this scheme for the transmission of knowledge, according to which divinely revealed wisdom, originating with Adam, was transmitted through the Chaldeans, Babyloneans, Persians, Egyptians, to come to full fruition with Plato, we find traces in the above quotation from Possevino. Ricci's mastery of the art of memory is another indication of Hermetic influences, as, since Frances Yates'work, memory-art and Hermeticism have become recognised as two sides of the same coin.[32] As Hermeticism tended to be a universal system of knowledge, we should be prepared to accept that mathematics was an important part of this system. There certainly does exist a "body of late sixteenth century speculations on the nature of 'universal mathematics' (*mathesis universalis*)".[33] However, the extent of these influences remains to be determined. Let it suffice, for our purpose, to note that mathematics, in the school of Clavius, had a taste of the divine. Still almost a century later, Pardies, the author of a very popular "short-and-easy" Euclid that would serve as the basis for the general section on geometry in the *Shuli jingyun* (1723),[34] would link mathematical reflection to reflection on God.[35]

A final reason for Ricci to translate Euclid, concerns an aspect of the work that above all has given it its fame and influence on general culture: its methodology. In the *Tianzhu shiyi* [The True Meaning of the Lord of Heaven] the Western scholar tries to convince his

29. Clavius, *In Sphaeram Joannis de Sacro Bosco Commentarius*, Rome, 1570. See further F.A. Homann, S.J., "Christopher Clavius and the Isoperimetric Problem", *Archivum Historicum Societatis Jesu*, XLIX (1980) pp. 245-254.

30. Matteo Ricci and Li Zhizao, *Yuanrong jiaoyi*. Included in *Tianxue chuhan*, repr. in *Zhonguo shixue congshu*, n° 23, vol. 6, pp. 3427-3482, Taibei, 1965. See J.-C. Martzloff, *Histoire des mathématiques chinoises*, Paris, 1988, p. 336 (Appendix).

31. *Coimbricenses, Commentarii Collegii Conimbricensis Societatis Iesu in octo libros Physicorum Aristotelis Stagiritae*, Cologne, 1602. First published at Coimbra, 1591. Reference in E. Grant, *Much Ado About Nothing. Theories of space and vacuum from the Middle Ages to the Scientific Revolution*, Cambridge, 1981, p. 347, n. 108.

32. Frances A. Yates, *The Art of Memory*, London, 1966. On Ricci's art of memory, see M. Lackner, *Das Vergessene Gedächtnis*, Stuttgart, 1986, a translation with commentary of the *Xiguo jifa*.

33. N. Jardine, "Epistemology of the sciences", in C.B. Schmitt and Q. Skinner ed., *The Cambridge History of Renaissance Philosophy* 1988, pp. 685-711, p. 705.

34. For an analysis of the structure of the mathematical encyclopedia of 1723 and its integration of Western and Chinese mathematics, see C. Jami, "Classification en mathématiques: la structure de l'encyclopédie *Yu Zhi Shu Li Jing Yun* (1723)", in *Revue d'Histoire des Sciences*, XLII-4 (1989), pp. 391-406. Pardies' work was: I.G. Pardies, *Élémens de Géométrie*, Paris, 1671.

35. I.G. Pardies, *op. cit.*, pp. 53-60.

hypothetical opponent via deductive reasoning of the necessary existence of God.[36] He even gives one of the famous scholastic proofs of His existence. But in his diaries he complains that the Chinese lack logic and are not sensitive to rational conversion strategies.[37] So Euclid could provide a "crash-course" in logic. This point of view is not very hard to understand if we consider that Euclid has been taught at secondary schools *ad nauseam* until quite recently with the objective of teaching children how to think and reason correctly. Also, already in the twelfth century we find examples of the transfer of Euclidian methodology into theology.[38] Of course the most famous example of this kind of phenomenon is Spinoza's ethics *de more geometrica*. In the seventeenth century axiomatics was again frequently used as a weapon against Scepticism.[39]

Clavius was one of the first to extend what is usually called the axiomatic (or axiomatic-deductive) method to applied mathematics, *in casu* gnomonics.[40] What is at stake here is a very complicated evolution of methodology that took place during the sixteenth and seventeenth centuries, by which the geometrical method was emancipated from a scientific methodology dominated by Aristotle's *Organon* (litt. "tools"), especially the *Posterior Analytics*. Method was an intensely debated issue during the sixteenth century, and in those discussions a major concern was the classification of sciences. I cannot go into this very deeply here, but still I would like to give an impression of the problems involved, because they show what difficulties had to be overcome to come to grips with the Euclidian "lucidity".

Let us start with the first proposition of the first Book of Euclid in which it is required to construct an equilateral triangle on a given line-segment, and to prove that the triangle thus constructed is indeed equilateral. Clavius shows in his commentary how the proof can be rewritten into syllogisms.[41] He then remarks that in principle the same can be done with other proofs, but that mathematicians do not proceed in that way, i.e. via syllogisms.[42] This attempt of his may serve to show what was a major concern to Renaissance academics: how to evaluate Euclid in a context dominated by Aristotle's *Posterior Analytics*. In this work, which is his most comprehensive and systematic description of scientific methodology, Aristotle had determined how, and under what conditions, true and certain knowledge

36. Matteo Ricci, *The True Meaning of the Lord of Heaven*, (trsl. D. Lancashire and P. Hu-kuo Chen), St. Louis, 1985.

37. See, for example, P. D'Elia, S.J., *Fonti Ricciane*, Rome, 1942-1949, vol. 1, p. 39, n. 55. Cf. J. Gernet, *Chine et christianisme*, Paris, 1982, p. 3.

38. See for example G.R. Evans, "The 'Sub-Euclidean' Geometry of the Earlier Middle Ages, up to the Mid-Twelfth Century", *Archive for History of Exact Sciences*, vol. 16 (1976-77), pp. 105-118, esp. pp. 113-114.

39. See H. Schüling, *Die Geschichte der Axiomatischen Methode im 16. und beginnenden 17. Jahrhundert (Wandlung der Wissenschaftsauffassung)*, Hildesheim-New York, 1969, p. 98: "Er [Jean-Bapt. Morin] leitet die Reihe jener im 17. Jahrhundert nicht seltener Versuche ein, die Existenz Gottes in axiomatisch-deduktiver Form zu beweisen."

40. *Ibidem*, p. 97. To my knowledge, this is the only study devoted exclusively to the fate of the geometrical method.

41. Clavius, *op. cit.*, p. 20.

42. Cf. N.W. Gilbert, *Renaissance Concepts of Method*, New York, 1960, p. 90, n. 38.

(*scientia*) is acquired.[43] At the basis of the system are some basic assumptions, or First Principles, that have to be accepted without proof, to avoid an infinite regress (ideally they are so evident that no one can doubt them). The instrument of gaining new knowledge is the demonstrative syllogism, consisting of a major premise (*major*), a minor premise (*minor*), and a conclusion. The *major* and the *minor* are linked via the middle term. The most perfect (*potissimae*) demonstrations are those that expose the proper, immediate, and commensurate cause of an effect. Of course "cause" is understood in the strict Aristotelean sense. Such a demonstration was a *propter quid* demonstration.

Now, in the *Posterior Analytics*, Aristotle himself held up geometry as the most perfect model of scientifically demonstrated knowledge, and he gives many geometrical examples. So, considering the admiration for Aristotle during the Middle Ages, it is no wonder that Euclid was studied in an Aristotelian framework. In fact, Clavius, according to his biographer, started his study of Euclid because of his reading of the *Posterior Analytics* during his stay in Coimbra.[44] Although during the Middle Ages Euclid was widely read and commented on, the focus was almost exclusively on its relevance to the all-important study of philosophy. As one modern scholar has succinctly put it: "it was something to be talked about".[45] There was a strong preoccupation with First Principles, the problems of the Infinite and Continuity, and the logical structure. Proofs were usually not given in full, but replaced by instructions for carrying out the actual proofs. The way of treatment was also adapted to the scholastic *quaestio*-form, introducing the objections of an adversary.[46]

Humanism, reacting strongly against medieval logic and methodology, had added many new sources to the "stock" of methods. Plato, Stoicism, and later Greek commentators on Aristotle gained influence, and also the views of the second century medical doctor Galen on method became very important.[47] Out of this "melting-pot" the conceptions of scientific method of the next century would take form, but methods pertaining to the arts and those pertaining to the acquisition of science were not clearly separated.[48] Finally, Proclus' commentary and the encyclopedic *Collectio Mathematica* of Pappus (late third

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43. For a good account see W.A. Wallace, *Galileo and his Sources. The Heritage of the Collegio Romano in Galileo's Science*, Princeton, N.J., 1984, Chapter 3 [Sciences and Demonstrative Methods].

44. Bernardino Baldi, *Vite de' Matematici*, edited by Guido Zaccagnini, *Bernardino Baldi nella vita e nelle opere*, 2nd ed., Pistoia, 1908, p. 335: "Alle matematiche cominciò Cristoforo ad attendere, come intesi da lui, con l'occasione degli studij della Posteriora d'Aristotile, perciocchè, essendo quel libro molto ricco d'essempij matematici, egli desideroso di ben intendergli si pose per sè stesso senz'altro aiuto di maestri ad affaticarvisi di maniera che in queste professiono egli afferma d'essere, come dicono i greci, autodidascalo." Cf. F.A. Homann, *op. cit.*, p. 238.

45. M.S. Mahoney, "Mathematics", p. 162. On the philosophical preoccupations of Medieval mathematics see also J.E. Murdoch, "The Medieval Euclid: Salient Aspects of the Translations of the *Elements* by Adelard of Bath and Campanus of Novara", in *Revue de Synthèse*, III^e Suppl. to n^os 49-52 (1968), pp. 67-94, and J.E. Murdoch, "The Medieval Language of Proportions: Elements of the Interaction with Greek Foundations and the Development of New Mathematical Techniques", in A.C. Crombie ed., *Scientific Change*, London, 1963, pp. 237-271.

46. H. Schüling, *op. cit.*, p. 110.

47. See, for example, N.W. Gilbert, *op. cit.*, *passim*.

48. *Ibid.* pp. xxii-xxv. Cf. H. Schüling, *op. cit.*, p. 110: "Wer in Wissenschaft und Gelehrsamkeit des 16. und des beginnenden 17. Jahrhunderts eindringt, gewinnt das Bild einer ungeheuren Vielfalt der Methodologien und der praktisch angewandten Verfahren bzw. Verfahrenskombinationen."

century AD) became available.[49] Euclid was put forward as an alternative to medieval logic, instead of a supplement to it, but at the same time the relation of Euclid to the *Posterior Analytics* was becoming more and more problematical, as was in general the place accorded to mathematics within the medieval scheme of the classification of sciences. The extent to which such problems occupied scholars may be gauged by a glance at the polemic on the certitude of mathematics that extended from the middle of the sixteenth century into the next one.[50]

The polemic took form with the *Commentarium de certitudine mathematicarum*, written in 1547 by the Siennese philosopher Alessandro Piccolomini (1508-1578).[51] In this work he sets out arguments to show that mathematical demonstrations fail to satisfy the Aristotelian conditions for *demonstrationes potissimae*. Because mathematics abstracts from matter and does not consider motion, it does not specify real causes, i.e. one of the four types of Aristotelian causes (material, formal, final, efficient). Also, since very often the same result may be proved by different demonstrations, it cannot be said that the only proper cause has been revealed. In denying mathematics the status of a true science, he was followed by the Jesuit theologian and natural philosopher Benito Pereira (1535-1610).[52] In a similar vein he argued that the subject-matter of mathematics is quantity, and that quantity is but an accident of matter. So mathematics does not occupy itself with essences. Towards the end of the century other writers (for example Celsus Manzinius, professor of moral philosophy) would stress that mathematical objects such as circles and lines do not exist independently, but only in the mind.[53] The Coimbra Jesuits, finally, also denied mathematics its true causes.[54]

On the other side were such as Hier. Balduinus and Jac. Scegk who equated the Euclidian method with the *Posterior Analytics*,[55] but more interesting were those who held an intermediate view.[56] F. Barozzi (the translator of Proclus), Barthol. Viotti (Prof. of Phil. and Medicine in Turin, 1568) and Jos. Blancanus (1566-1624), a pupil of Clavius, would maintain that, although the method of geometry does not conform to the Aristotelian norm, it still is a science.

To give a more tangible impression of the way the problem was discussed, let me refer to the example given above, the construction of the equilateral triangle of the first proposition of Book I. The construction is brought about by drawing two circles with equal diameters, and it is then proved that the constructed triangle is equilateral by pointing out that the triangle's three sides are rays of equal circles, and so are equal to each other. Piccolomini would say that this is not an Aristotelian demonstration, because properties of

49. The *Collectio* of Pappus was translated in 1589 by Commandino and owes much of its importance to its discussion of the method of analysis. It was to become very important for the development of algebra and I will discuss it towards the end of this paper.

50. I base the following discussion above all on H. Schüling, *op. cit.*, pp. 45-55.

51. *Ibid.*, p. 45.

52. *Ibid.*, p. 47.

53. *Ibid.*, p. 49.

54. *Ibid.*, p. 50.

55. *Ibid.*, p. 51.

56. *Ibid.*, pp. 52-55.

circles are used to prove something about triangles, and how can properties of circles be the cause of a property of a triangle? Or, to prove that the sum of the angles of any triangle is equal to two right angles, Euclid uses an exterior angle of the triangle. How can something exterior to the triangle be considered to be the direct cause of a characteristic interior to the triangle?

The struggle over the *mos geometricus* was only part of a much wider concern about method during the period under consideration. Research into this area has only quite recently been taken up seriously, and it turns out that a tremendous diversity of methods and ideas about method were being discussed during the sixteenth century. There was a general tendency to give more prominence to reasoning that is concerned with only probable arguments, as is found in dialectics and rhetorics.[57]

I have two reasons for mentioning these scholarly debates. In the first place they should serve as a warning that mathematics was embedded in an intellectual milieu totally different from that of the modern world. We cannot say that the presence of Greek mathematics led to the Scientific Revolution in a direct way. If we talk about the axiomatic method or deductive thinking, we should be continuously aware of the very different perceptions of sixteenth century thinkers. We can only conclude rather confidently that method was indeed intensely debated, which in itself is a major difference with Chinese culture.

My second reason for touching upon sixteenth century methodology is more directly linked to the translation of Euclid. For example, in the Ricci-Xu translation we find ourselves confronted with the enigmatic translation of "demonstration" as *lun*. Now, it can hardly be said that *lun*, which, after all, means more something like "discussion", conveys anything like our modern concept of "demonstration". Almost to the contrary, because it is the very characteristic of a rigorous proof that it doesn't allow discussion. I do not pretend to be able to resolve this question here, but I would like to suggest that we have to take into consideration contemporary ideas on method, in particular the transfer of methods derived from dialectics and rhetoric to other domains. For example, Commandino, in the *Prolegomena* to his translation of Euclid (1572), tried to show that Euclid used Plato's four dialectical methods in his proofs.[58] Also, according to Averroes, the foundations of geometry were *propositiones opinabiles*.[59] Further on in this paper I will propose another explanation. To conclude this discussion of Euclidian method, I would like to cite the title of a book written by Guillaume Postel in 1543, to illustrate a belief in the converting power of Euclid. The book was written for "Jews, Mohammedans and Heathens, who can't be persuaded by Authority alone": *Sacrarum Apodixeon, seu Euclidis christiani, libri ii*.[60]

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57. Cf. *The Cambridge History of Renaissance Philosophy*, p. 707: "A further sixteenth century development with important, but as yet little explored, epistemological implications is that of humanist promotion of the use of 'dialectical' methods of presentation and persuasion that concentrate on probable argument, argument by analogy, citation of exempla and so on rather than demonstration in the strict Aristotelian sense." Cf. N.W. Gilbert, *op. cit., passim*.

58. See F.A. Homann, *op. cit.*, pp. 233-246, esp. pp. 240-241, where he also gives a translation of the relevant passage.

59. H. Schüling, *op. cit.*, p. 49. It is noteworthy that Ricci's translation of "axiom", *gonglun*, has the meaning of "public opinion".

60. *Ibid.*, p. 93.

We have seen above, that not all Jesuits had a very high opinion of mathematics. In fact, the discussions on the certitude of mathematics led Clavius to write the following in his *Modus quo disciplinae mathematicae in scholis Societatis possent promoveri*:

> It will also contribute much to this if the teachers of philosophy abstained from those questions which do not help in the understanding of natural things and very much detract from the authority of the mathematical disciplines in the eyes of the students, such as those in which they teach that mathematical sciences are not sciences, do not have demonstrations, abstract from being and the good, etc.[61]

In the same pamphlet he complains that teachers of mathematics did not have the same status as their colleagues and were not allowed to take part in official disputations. He pleads strongly for a promotion of mathematics, and gives several reasons why mathematics is essential for the members of the Society of Jesus. He mentions its absolute necessity for understanding philosophy; the prominence it might give to the members of the Order; the harm that might be caused by blundering in the prestigious public debates on account of mathematical ignorance; the importance of mathematics to astronomy, calendar-making, etc.

The efforts of Clavius were a crucial impulse for what was after all a religious organization to take up a major enterprise in secular studies. It is no exaggeration to say that his promoting activities turned out to be very successful. Not very long afterwards it was generally acknowledged — even by their most severe critics — that some of the Jesuit colleges offered the best training in mathematics and science available.

Quite early after their foundation — though not right from the start — the Jesuits took up an educational mission as an important way to reach their goals.[62] By educating both the clergy and the children of the nobility they would gain the influence they wanted. Often in a competitive way, they took over communal schools, founded special boarding schools for sons of princes, and to crown their growing body of instruction they had such show-pieces of academic learning as the Collegio Romano and a few other universities.[63] Somehow they had to legitimize such secular, non-contemplative intellectual activity. They did develop some kind of an "ideology of knowledge as a path to salvation".[64] But slogans like Pereira's "amicus Socrates, amicus Plato, sed magis amica veritas" showed the danger of a secular search for knowledge.[65] A result of the tension this created, both within the Society and in its relation to the Catholic Church, was the famous *Ratio Studiorum* (final version 1599 after some preliminary drafts) as an instrument for imposing conformity. The Jesuits' ultimate

61. Again, I took the translation from A.C. Crombie, *op. cit.*, p. 66. The text may be found in *Monumenta Paedagogica Societatis Jesu quae Primam Rationem Studiorum anno 1586 praecessere*, Madrid, 1901, pp. 471-474.

62. See for example A. Scaglione, *The Liberal Arts and the Jesuit College System*, Amsterdam-Philadelphia, 1986, p. 2 *et passim*.

63. P.F. Grendler, *Schooling in Renaissance Italy. Literacy and Learning, 1300-1600*, Baltimore, 1989, pp 363-376.

64. See R. Feldhay, "Knowledge and Salvation in Jesuit Culture", in *Science in Context*, 1, 2 (1987), pp. 195-213, esp. p. 200.

65. *Ibid.*, p. 201.

failure to reconcile science and religion is responsible for the ambivalent position they occupy in the history of science.

Ever since the role they played in the Galileo lawsuit, the Jesuits have been identified with arch-conservatism. During the last few decades, however, their contribution to science has been reassessed. On the one hand, adherents to the "continuity thesis" in the history of science have singled them out as intermediaries in the transmission and transformation of important medieval concepts and preoccupations. For example, Wallace, on the basis of hitherto neglected notebooks of Galileo, has tried to show that he was deeply influenced by lectures given at the Collegio Romano, and that his struggle to find a satisfactory experimental methodolgy for his science of motion was very much determined by Jesuit methodology (especially the concept of *ex suppositione* reasoning).[66] Also, their importance in giving a major place to experiments has been stressed.[67]

Furthermore, it is quite clear that they made important contributions to some branches of science. For example, in a book on the history of electricity and magnetism the author writes: "knowledge about electricity was kept alive during the seventeenth century by Jesuit polymaths" and "they also enriched the subject with valuable observations".[68] Also mechanics, optics and observational astronomy, in which they were greatly helped by having observers in many different parts of the globe, may be mentioned. Other rather modern aspects are: some very well equiped colleges as far as instruments are concerned and the building and maintenance of those instruments[69]; the institution of research periods for their more distinguished members, freeing them from teaching obligations for some years and encouraging them to write textbooks;[70] the Collegio Romano as maybe the first "Scientific Academy";[71] and a comprehensive system of international correspondence.[72]

Clavius corresponded with many of the major scientific personalities of the period. His contributions to mathematics are not very spectacular as far as originality is concerned, but he was definitely a great catalyst of mathematical activity, both as a teacher and through his great erudition. The number of works he wrote is impressive, covering practically the whole field of contemporary mathematics, and Knobloch has counted about 140 astronomers and mathematicians that he cited in his editions.[73] He was thus an exemplary

66. See W.A. Wallace, *op. cit.*

67. For example P. Dear, *op.cit.*, pp. 133-175.

68. J.L. Heilbron, *Electricity in the 17th and 18th Centuries. A Study of Early Modern Physics*, Berkeley, 1979, p. 101.

69. *Ibid.*, p. 103.

70. *Ibid.*, pp. 105-106.

71. See U. Baldini, *op. cit.*. This very well-documented paper shows Clavius as the founder of an "Academy of Mathematics" that had materialised around 1600. There exists a document by Clavius containing a project for the creation of special research institutions. For further reference see pp. 143, 144 and p. 160 n. 39 of the above-mentioned article.

72. See for example E.C. Phillips, "The correspondence of Father Christopher Clavius S.J. preserved in the Archives of the Pont. Gregorian University", *Archivum Historicum Societatis Iesu*, VIII (1939), pp. 193-222, p. 194.

73. E. Knobloch, "Sur la vie et l'oeuvre de Christophore Clavius (1538-1612)", *Revue d'Histoire des Sciences*, XLI-3/4 (1988), pp. 331-356.

Renaissance personality in his synthetic efforts to integrate the classical mathematical heritage with the rapid developments of his own century.

Let us now turn to his version of Euclid. In his preface Clavius makes it clear that his main objective is to be a guide to students. Another, rather casual, remark he makes in this preface may serve as a warning for the kind of problems we have to face in placing the work in its historical context.[74] For he remarks that the demonstrations of Theon are, in his opinion, actually those of Euclid. Now Theon of Alexandria (4th century A.D.) had made a redaction of Euclid that was to remain the basis for all editions of Euclid up to the end of the nineteenth century, i.e. until the discovery by Peyrard of an older redaction in the Vatican Library.[75] If we turn to the 1572 translation by Federigo Commandino, we find in his *Prolegomena* a rather long passage in which he gives arguments that Theon's proofs are in fact Euclid's own proofs.[76] Obviously, until 1572 it was believed that the proofs accompanying the theorems were not written by Euclid himself, but were added by later writers as a kind of commentary. In editions of the beginning of the sixteenth century we find the commentary of Campanus contrasted with the commentary by Theon.[77] This disconnection of proofs from theorems is rather surprising to say the least, as it leads to the conclusion that proofs were not considered essential to the work. We might also look here for an alternative explanation for Ricci's translation of "demonstration" by *lun*. Proofs were actually referred to by such words as *commentarius* and *expositio*, but Clavius himself speaks about "*demonstrationes*".[78]

Both Clavius' and Commandino's editions were the first to show a clear sense of the complex textual transmission of Euclid. In a beautiful book, Rose has described the Humanists' efforts at restoring the Greek mathematical texts. He has given a vivid picture of the manuscript-hunting, the fate of the individual manuscripts, many of which were brought over from Byzantium, and the many impressive printing projects for the entire Greek *Corpus Matheseos*. But at the same time, Clavius' edition retains many medieval aspects. To appreciate the place of his version it is necessary to give a minimal account of the fate of Euclid in Western Europe.[79]

Euclid had never been completely eclipsed. Fragments of the *Elements* — axioms, postulates, definitions, and most of the enunciations of the first four Books — had survived in a translation made by Boethius in the sixth century. They appeared mostly appended to

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74. "Demonstrationes aliorum, maximè Theonis, quas quidem ipsius esse Euclides, non levibus argumentis adducti cum plerisque asseveramus, et Proclus etiam testatur, breviores, quantum per rei difficultatem licuit, vel certè planiores, quando illud non potuimus, dilucidioresque reddere conati sumus." Clavius, *op. cit.*, *Praefatio*. This citation may also serve to illustrate that he adapted many proofs.

75. See J.E. Murdoch, "Euclid: Transmission".

76. His conclusion is: "Sunt igitur ille quidem demonstrationes Euclides, sed eo modo conscriptae, quo olim Theon Euclidem secutus suis discipulis explicant."

77. Marshall Clagett: "The Medieval Latin Translations from the Arabic of the *Elements* of Euclid, with Special Emphasis on the Versions of Adelard of Bath", *Isis*, 44 (1953), pp. 16-42, p. 19.

78. At least in the second, 1589, edition. As mentioned before, I have not been able to consult the first edition, which was, in all probability, the one used by Ricci. For example, in the *Jihe yuanben* we find the famous parallel postulate as the eleventh axiom. Clavius explains in the second edition (in a note to the reader) that this postulate is the thirteenth axiom in this edition whereas it was the eleventh in the first. Still, D'Elia seems to believe that Ricci used the second edition (P. D'Elia, "Presentazione della prima traduzione cinese di Euclide", p. 188, n. 68). A fuller discussion will be presented in my Ph.D. dissertation.

79. J.E. Murdoch, "Euclid: Transmission".

manuals of mensuration. Also, knowledge of practical geometry had been kept alive by practitioners of all sorts. However, in the eleventh century familiarity with Euclid was still at such a low level that Paul Tannery has remarked that the history of geometry in the eleventh century is above all a history of ignorance.[80] Although not very many documents have survived, he cites by way of example a correspondence between a certain Ragimboldus, *scholasticus* (head of a cathedral school) of Cologne, and Radulfus of Liège, in which they discuss what might be meant by an "interior angle".[81] Tannery concludes that even men used to reason and calculate did not make anything of theoretical geometry.[82]

This situation began to change during the first half of the twelfth century with the appearance of the translations from the Arabic in the south of Spain. It would still take a long time before Euclid began to be understood, but from that time on there was a growing stream of translations, commentaries and redactions. By far the most influential translation would become the so-called Adelard II translation, together with the translation by Campanus of Novarra (13th century) that was based on the Adelard II.[83] These translations were not full translations however. Clagett calls them "abridgements"[84] and on the whole they do not contain complete proofs. A general characteristic of the medieval Euclid is a tendency to render the work more suitable as a school book. The reader is often addressed in the second person, and he receives instructions on how to achieve constructions via practical advice and on how to carry out proofs. Axioms and postulates are added abundantly to fill up any gaps in the reasoning. What was certainly an advantage over the Greek text was the appearance of cross-references in the form of indications of what axiom, definition or previous theorem is being used at each step, a feature that is completely absent from the original and that was preserved by Clavius.

Murdoch[85] has indicated four events that mark the transition to the Renaissance Euclid: the publication of the first printed Euclid (the thirteenth century translation by Campanus) in 1482; the first translation directly from the Greek, published in Venice in 1505, by Zamberti, who refers to Campanus as *ille interpres barbarissimus*;[86] the 1533 *editio princeps* of the Greek text; the translation by Commandino in 1572. The first half of the sixteenth century was the scene of a continuing quarrel between defenders of the

80. "Ceci n'est pas un chapitre de l'histoire de la science; c'est une étude sur l'ignorance...", in Paul Tannery's *Mémoires Scientifiques*. Tome V: *Sciences exactes au Moyen Age*, Toulouse-Paris, 1922, p. 79. I am grateful to Karine Chemla for drawing my attention to those misunderstandings.

81. *Ibid.*, pp. 90 ff.

82. *Op. cit.*, p. 93: "Ainsi, même en possédant l'énoncé d'un des théorèmes les plus élémentaires de la géométrie plane, les maîtres les plus renommés du onzième siècle sont incapables d'en comprendre exactement le sens, et ils échouent plus ou moins en essayant de le démontrer. On peut juger, par cet exemple, combien il était difficile en fait de constituer la géométrie théorique, même pour des hommes exercés à raisonner et à calculer."

83. J.E. Murdoch, "The Medieval Euclid", p. 69: "Adelard's Euclid was the fountainhead for the really substantial multiplication of versions we have referred to above." This very lucid article is fundamental for an understanding of the Medieval Euclid.

84. M. Clagett, *op. cit.*, pp. 20 ff.

85. J.E. Murdoch, "Euclid: Transmission", p. 448.

86. P.L. Rose, *op. cit.*, p. 51.

Campanus translation and adherents of the Zamberti translation[87] which were often published together. As an illustration of the lack of awareness of the historical transmission of the text it is interesting to note that Zamberti, while fiercely criticising Campanus, was unaware of the fact that Campanus had translated from the Arabic, although Campanus is full of transliterations from the Arabic.[88]

Commandino's translation remained a standard into the nineteenth century. Clavius' work is different in character. Without being exhaustive, I would like to note some of the features of his version (in addition to those already mentioned, such as his body of cross-references). Much of the difference lies in its didactic focus. The most evident difference is the enormous amount of commentaries. He adduces many alternative proofs from other sources, and gives frequent practical instructions on how to carry out constructions in the easiest and most convenient way. Here he is clearly addressing those who actually have to use geometry. The disadvantage of this approach is that someone who does not understand the ideal nature of the constructions in Euclid — limited to those that can be completed with straight-edge and compass as sole instruments — might easily miss the point. Why does Euclid not give the simplest construction?

Next to be mentioned are his many additions to the body of the text. He has added many theorems, especially at the end of Book VI, but also axioms. The addition of axioms is the continuation of a medieval development. On the one hand it shows a clear appreciation of the deductive structure of the work and the wish to fill gaps in the reasoning — the insertion of an axiom of continuity is especially to be admired,[89] although it was already quite general during the Middle Ages to add such an axiom —, but on the other hand he sins against the requirement to keep the number of basic assumptions to a minimum. Clavius also occupies a modest place in the history of the development of non-Euclidian geometry because of his attempt in the second edition to prove the famous postulate of parallels, but this proof didn't find its way into the *Jihe yuanben*.[90]

By far the most noteworthy aspect of the Clavius edition is the commentary in which he either discusses contemporary disputes, or presents long treatises on special subjects. Thus it happens that the *Jihe yuanben* contains a discussion of the long-standing controversy over the nature of so-called horn-angles.[91] These angles — formed by a straight line and a curved line (e.g. the circumference of a circle) — appear in theorem 16 of Book III, and had already vexed medieval commentators. Clavius was engaged in a vehement polemic over these angles with Jacques Péletier, a Parisian translator of Euclid. The problem was to decide whether those angles were to be accepted as "homogeneous quantities", and thus be allowed to form a ratio with angles formed by straight lines only. Although this problem played a certain role in the development of calculus, the whole

87. Tartaglia, author of the first translation into a modern language, subtitled this translation of 1543 as "Euclid, according to the two traditions", by which he meant Campanus and Zamberti.

88. H. Weissenborn, *Die Übersetzungen des Euclid durch Campano und Zamberti*, Halle a/S., 1882, p. 25.

89. The fourth postulate guarantees the existence of a fourth proportional. This postulate had already been added by Campanus.

90. It was only added in the second edition (and later).

91. For a full discussion, see L. Maierù, " '... in Christophorum Clavium de Contactu Linearum Apologia'. Considerazioni attorno alla polemica fra Péletier e Clavio circa l'angolo di contatto (1579-1589)", in *Archive for History of Exact Sciences*, XLI (1991), pp. 115-137.

controversy carries a distinctly scholastic flavour. It certainly must have presented a mystery to Chinese readers.

Those horn-angles have brought us to the fifth Book of the *Elements*, because the fourth definition[92] of this Book tells us what magnitudes may be compared to each other and thus form a ratio: "Magnitudes are said to have a ratio to one another which are capable, when multiplied, of exceeding one another."[93] They should be homogeneous (lines with lines, surfaces with surfaces etc.), and, by a suitable multiplication, be capable of exceeding each other (Any two magnitudes A and B, where $A<B$, can be said to have a ratio to one another, if it is possible to find a number n, such that $nA>B$). This is a rather important definition for the history of science, because its effect is the exclusion of the infinitely large and the infinitely small from the domain of ratio, thus forming an obstacle to the use of infinitesimals. The subject of the fifth Book is ratio and proportion between magnitudes. Inserted between the definitions of this Book, we find one of the special treatises of Clavius: a discussion of different kinds of ratios, and arithmetic, geometric and harmonic proportions[94]. This discussion of ratios is essentially a classification of fractions, that was very common during the Middle Ages and that harks back to the *Introductio Arithmetica* of Nicomachus of Gerasa who flourished about 100 A.D.[95] Ratios are classified according to whether the "numerator" is greater than, equal to, or smaller than the "denominator". To a modern reader this classification is rather useless, because we don't make any distinction between ratios of numbers and fractions. However, as the Greeks did not accept fractions as numbers in their own right, they were forced to fall back upon ratios. Whereas a fraction is just one number, a ratio is of necessity a relation between two numbers (or magnitudes). In the time of Clavius the step to treat ratios between numbers as fractions had not been fully made. What is even more remarkable, though, is the appearance of this treatise at the beginning of Book V.

Two Books of the *Elements* deal with ratio and proportion: Book V and Book VII. Book VII considers ratio and proportion between numbers. Book V has as subject magnitude in general, but it is obviously aimed at geometrical magnitudes (lines, angles, etc). We are witness here to a fundamental characteristic of Greek geometry in general, that has caused great problems of understanding for its heirs: the rigid separation between number and magnitude. The deep gap between the two had necessitated a dual treatment of ratio: one in the "language" of number, and one in the "language" of magnitude. The reconstruction of Greek mathematics prior to the *Elements* is still a much debated issue, but most historians of mathematics agree on the following schematic "scenario".[96]

The existence of the two Books reflects the heterogeneous provenance of the *Elements* and they are the fruit of different stages in the evolution of the *Elements*. Book VII is undoubtedly the oldest and contains Pythagorean material. As is well known, for the

92. That is definition 5 in the *Jihe yuanben*, p. 765.

93. This, and all following passages from the Greek of Euclid's Elements, are from the translation by Sir Thomas Heath, *Euclid. The Thirteen Books of the Elements*, (3 vols., Cambridge, 1926; reprinted New York, 1956).

94. Clavius, *op. cit.*, pp. 195-244. *Jihe yuanben*, pp.760-764.

95. See Sir T. Heath, *A History of Greek Mathematics*, 2 vol., New York, 1981 (Reprint of the 1921 edition), vol. 1, pp. 97-112.

96. For a lucid treatment, see B.L. van der Waerden, *Science Awakening* (English translation of *Ontwakende Wetenschap*), Groningen, 1954, chapters 5 and 6.

Pythagorean sect numbers were of great mystical and cosmological significance. Their doctrine of "all is number" entailed that all things and phenomena can be expressed in numbers. "Numbers" for them meant positive integers. They — as well as all Greek mathematicians and philosophers — did not accept fractions and irrational numbers as numbers in their own right. It was a fundamental tenet that the unit cannot de divided. At the same time they had started to develop mathematics as a rigorous system, demanding deductive proofs. At some point it was discovered and proved that there exist relations between things that cannot be expressed as relations between whole numbers. The canonical example is the ratio between the side and the diagonal of a square, also mentioned by Ricci at the beginning of Book V. The consequence of this discovery was that whole numbers could no longer serve as a basis upon which the whole edifice of mathematics could be built. It proved that there exist magnitudes that cannot be measured (after a unit measure has been chosen), that is, expressed in whole numbers, or ratios between whole numbers. The older proportion theory of Book VII was based on the following definition of proportional numbers: "Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth." This definition only makes sense if it is possible to find a common measure that "measures" both members of a ratio, or, in other words, between commensurable magnitudes. Whole numbers are always commensurable with one another, because there is always the unit that measures both. But it is clear that the above definition does not cover geometrical magnitudes in general, and leaves many "holes". When dealing with geometrical entities other building blocks had to be devised.[97] The remedying of the gap caused by the discovery of the irrational is generally attributed to Eudoxus of Knidos, a member of the Academy of Plato. The foundation of his reformulation of proportion theory for geometric magnitude is the famous fifth definition of Book V[98] which gives a general criterion for the equality of ratios, no matter whether they are between commensurable or incommensurable magnitudes:

> Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and the fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

(Symbolically: $A/B = C/D$ if, and only if, for all integers m, n, when $nA > mB$ then $nC > mD$; when $nA = mB$ then $nC = mD$; and when $nA < mB$ then $nC < mD$.)

By many authors the Eudoxean definition has been considered as one of the most beautiful and subtle creations of Greek mathematics. At the same time it has been remarked that it was a direct result of the limited number-concept of the Greeks: because they rejected irrational numbers, they could not express incommensurable ratios directly as numbers. But the discovery of irrationality would have been unthinkable without the search for exactitude and high standards of rigor, for which the Greeks are unique. On the other hand, the strict

97. Or, when solving algebraic equations (B.L. van der Waerden, *op. cit.*, p. 126), but it is a hotly debated issue whether parts of the geometry of the Greeks were actually algebra. This is the problem of "geometric algebra", i.e. algebra in geometrical "clothes".

98. Definition 6 in the *Jihe yuanben*, p. 767.

separation between number and magnitude — the iron consequence of this rigor — in many ways greatly hindered the further development of mathematics. It was for men like Viète, Fermat and Descartes to break down this barrier definitely, and venture freely into the further "arithmetisation" of mathematics, recoupling number and magnitude.

What concern us more here at the moment are the problems of understanding that manifested themselves. Already the Arabic commentators had tried to reinterpret definition 5, imposing on it an arithmetical interpretation. There exist numerous Arabic commentaries on Book V.[99] We have to realize that Euclid left no explanations whatsoever, and the reconstruction sketched above has been the result of many centuries of interpretative efforts. The fate of definition 5 in Medieval Europe was in addition complicated by a mistranslation. The Adelard II and Campanus versions contain a spurious definition 4 — substituting a definition of continuous proportion — which has been partly responsible for the misunderstanding of Book V.[100] Murdoch even speaks of an almost complete failure to understand the Eudoxean definition. The outcome of this misunderstanding was the replacement of the Eudoxean criterion for the equality of ratios by an alternative criterion: that of equality by "denomination". Those ratios were declared equal, whose denominations are equal.[101] The problem of what was to be understood by "denomination" was never completely solved, but the concept was used with great ingenuity by men such as Thomas Bradwardine and Nicole Oresme. It definitely is a numerical concept: it assigns numbers to ratios. In the case of commensurate ratios the idea is straightforward. For example, the ratio between 6 and 2 is denominated by 3 because 6 is 3 times greater than 2. But the problems start with incommensurable ratios. A frequent strategy was just to ignore them. According to Campanus, numbers are closer to the intellect, and it was the disease (*malatia*) of incommensurability that obscured the principles of Book V.[102]

We find this denomination concept present in the *Jihe yuanben* as the *mingshu* of a ratio: the number naming the ratio. It is introduced after definition 5 of Book VI.[103] This definition is almost certainly a later interpolation and it speaks of the composition of ratios: "A ratio is said to be compounded of ratios when the sizes of the ratios multiplied together make some (? ratio, or size)". As can been seen here, Heath is not able to make sense of the definition and remarks that the multiplication of sizes is an operation unknown to geometry.[104] The "size" of the ratio has been rendered by Clavius as "denominator". The definition does not appear in Campanus' translation, and its reappearance in the works of

99. For these Arabic commentaries, see E.B. Plooij, *Euclid's Conception of Ratio and his Definition of Proportional Magnitudes as Criticised by Arabian Commentators*, Rotterdam, 1950.

100. The definition is: "Quantitates que dicuntur continuam proportionalitatem habere, sunt quarum eque multiplicia aut equa sunt aut eque sibi sine interruptione addunt aut minuunt". For the definition, and for a brilliant discussion of the fate of Eudoxus' definition, see J.E. Murdoch, "The Medieval Language of Proportions". This mistranslation had subsequently influenced the interpretation of definition 5. In the following I rely heavily on Murdoch's paper.

101. The source for this replacement was probably Campanus, who replaced the usual definition of proportional numbers (Book VII, definition 20) with: "Similes sive una alii eadem dicuntur proportiones que eandem denominationem recipiunt". He, in turn, had taken this definition from Jordanus de Nemorare's *Arithmetica* (Book II, definition 9). I am almost literally quoting Murdoch, *Salient Aspects*, p. 80, n. 41.

102. J.E. Murdoch, *Salient Aspects*, pp. 88-89.

103. *Jihe yuanben*, p. 829.

104. T. Heath, *The Thirteen Books*, vol. 2, p. 190.

Commandino and Clavius is probably due to the importance of the composition of ratios in astronomy and practical applications. Ricci devotes a long commentary to the explanation of what is meant by the procedure of compounding ratios. It is rather bewildering for a modern reader, because what he tries to explain boils down to the multiplication of fractions, but the step is never fully made. Even modern commentators find it hard to decide whether the compounding of ratios was in fact a Greek analogue of multiplying fractions.[105]

Ricci explains that if we have two ratios A/B and B/C with a common middle term (*zhonglü*) B, the ratio A/C is composed of the ratios A/B and B/C in the sense that the *mingshu* of A/C is acquired by multiplying and/or dividing the *mingshu* of A/B and that of B/C. The middle term serves as the glue that "glues" together — or the button that holds together — the two ratios. If, on the other hand, we have two ratios without a common middle term, say A/B and C/D, we have to transform them into two other ratios with a common middle term before we can link them together (*xiangjie*) by using the middle term as glue. We have to find three new quantities E,F, and G, such that A/B=E/F and C/D=F/G. Now we can bind together E/F and F/G. Ricci calls this procedure very poetically the "technique of borrowing images" (*jie xiang zhi shu*). He also explains that the "double ratio" and the "triple ratio", which are defined in Definitions 10 and 11 of Book V,[106] are just special cases of this composition. If we have A/B=B/C, then A/C is called the double ratio of the ratio A/B (analogously for the triple ratio).

It is interesting to note that Ricci, at this point, makes one of his references to Chinese mathematics. He says that if we compare lines we need only the simple ratio; if we compare surfaces, we need the double ratio; if we compare volumes we need the triple ratio; and we need the quadruple and higher ratios in the cases where the mathematicians (*shujia*) perform the operation of taking the fourth — and higher — powers (*san chengfang*, *si chengfang* etc.).[107] What he means is that, if for example, we have two squares with sides of 3 units of length and 1 unit of length respectively, and we want to compare their size, then we have to square the ratio of their sides to find the ratio of their surfaces. For volumes we have to raise to the third power. But then he has "run out of his dimensions" and has to have recourse to Chinese terminology. This rather casual remark shows the ambivalent situation he finds himself in: he tries to interpret Euclid in a practical sense, but the text does not allow him to do so. He has hit upon one of the stumbling blocks of Greek mathematics.

For the important applications of geometry, such as measurement, it has to be "arithmetised". A unit measure has to be chosen, and lenghts have to be expressed in terms of this unit. We get the measure of a rectangle if we multiply two numbers (the sides of the rectangle), and of a rectangular parallelepiped if we multiply together three numbers. That is why the second power is called "square" and the third power "cube". Now, the development of modern algebra in Europe began with an attempt to translate geometrical problems into a form that lent itself to calculation: set up equations involving one or more unknowns that

105. I. Mueller, *Philosophy of Mathematics and Deductive Structure in Euclid's* Elements, Cambridge Mass., 1981, p. 88: "It is possible to say that compounding ratios is an analogue of multiplying fractions. The serious question for interpretation is whether compounding should be viewed as a representation of multiplying, i.e., as a device for representing the multiplication of fractions in the language of proportionality. I shall be arguing that compounding should not be viewed in this way." Cf. D.H. Fowler, *The Mathematics of Plato's Academy. A New Reconstruction*, Oxford, 1987, pp. 138-143. In chapter 7 of this book Fowler even denies the Greek any familiarity with common fractions.

106. Combined into definition 10 in the *Jihe yuanben* (p. 772).

107. *Jihe yuanben*, p. 773.

could be added, multiplied, raised to powers, and subjected to root extraction.[108] One of the fundamental problems that had to be overcome was the dimensional aspect of multiplication. What significance was to be attached to the unknown raised to a higher power than the third? The unwieldy names given to these higher powers, such as *solidiquadratus* for the tenth power, are an indication of the nature of the problem. Descartes would get rid of the problem in his *Géométrie* (1637) by defining multiplication in a different way, using proportion. In Ricci's passage cited above, it is as if we hear Descartes complaining that "such names ['square', 'cube', etc.] are to be altogether rejected, lest they confuse thought...".[109] He was concerned with problems that did not exist in Chinese mathematics.

I hope it has become clear that both the denomination concept and the classification of ratios forced an arithmetical interpretation on the general theory of ratio that is at odds with the original purpose of the book. In fact, the whole meaning of the fifth definition is lost within an arithmetical context. It is true that Ricci mentions the existence of incommensurable ratios (*xiao he*, small harmony), but the reader is referred to the non-translated Book X for a further treatment. Given the tremendous linguistic problem of translating this definition, he succeeds quite well in rendering the meaning of the words. He does however invert antecedence and consequent, and has not found an equivalent for "equimultiple", but the commentary makes quite clear what is intended by the definition. What the commentary leaves completely in the dark is the motivation for this definition and how it ever may be used in, for example, a proof.[110] On the contrary, by giving a numerical example, he only shows implicitly how unfit this definition is for numerical ratios: in order to show that 4/2=8/4 one has to test an infinite number of different cases.

Of course Ricci cannot be blamed for his failure of understanding. It has generally been acknowledged that Isaac Barrow in the next century was the first to have fully penetrated the mysteries of this subtle definition. The fact remains that, removed from its background of incommensurability, the definition falls flat. Moreover, the commentary he offers, though an admirable piece of translation, introduces more questions than answers. For one thing it doesn't give a hint what the *mingshu* of an incommensurable ratio might be. Secondly, he never actually takes the step of treating ratios as fractions and thus of simplifying the operations involving ratios. For Chinese mathematicians, who were very well used to treating ratios as fractions and manipulating them easily, this must have been very confusing. Ultimately, it is the Greek taboo on the extension of the number-concept to include fractions and irrational numbers that is responsible for the confusion.

In the seventeenth century the mathematician John Wallis would remark that "all of Book V is nothing but arithmetic".[111] How can anybody possibly make this claim when in the whole of Books V and VI there does not appear a single number? Newton, on the other hand, in 1705 still upheld that geometry and arithmetic had to be kept strictly apart, and that

108. See for example M.S. Mahoney, *The Mathematical Career of Pierre de Fermat*, Princeton, 1973, chap. 3.

109. *Regulae ad directionem ingenii*, Regula xvi. I took the quotation from M.S. Mahoney, *The Mathematical Career of Pierre de Fermat*, p. 43.

110. The second edition of Clavius does contain a discussion of Euclid's motivation: "Cur Euclides in Def. vi et vii quatuor magnitudines proportionales, et non proportionales per earum aeque multiplicia definierit.", pp. 249-251.

111. Citation taken from J.E. Murdoch, "The Medieval Language of Proportions", p. 271.

the geometry of the ancients should be kept "pure and uncontaminated" by numbers.[112] So, according to one great mathematician the "magnitudes" of Book V were just the "ghosts of numbers that have departed", according to the other, numbers should be kept in their proper "box". Both had radically different answers to the riddles posed by Greek mathematics. Even modern historians of mathematics offer very different interpretations of the peculiarities of Greek mathematics, and attempts at reconstructing its evolution yield wildly diverging points of view. Famous controversies over the "foundation-crisis" caused by the discovery of the irrational, the problem of "geometric algebra", the reason for the restriction of constructions to those by ruler and compass only, over whether the Greeks actually knew and used common fractions, over whether the deductive method and its terminology actually originated within dialectics or the other way round, over what exactly was the method of analysis: all these controversies show how difficult it is to evaluate Greek mathematics and appreciate its shortcomings and genius.

In his book *Science Awakening* van der Waerden has made some interesting remarks on the decline of mathematics in the Hellenistic world. As an important factor he singles out the interruption of the oral tradition.[113] Without oral explanations, without anybody pointing to the figures and indicating the important relations, the Greek mathematical works with their rigorous and monotonous structure, and long, exclusively verbal formulations, had become quite obscure. What was worse, they did not give any heuristic indication whatsoever and did not leave a trace or hint of how the results had been attained. I do not know whether van der Waerden is right in suggesting a causal link. I only use his remarks to draw attention to the difficulty, one might even say the unnaturalness, of Greek mathematical works. We have to realise that their rigid form is highly artificial. Used as we are to "proofs", "theorems", and "axioms", after the Greek style and form of presentation have set the standard, we should not forget that this form is conventional and does not mirror mathematical practice.

It is often said that the essence of mathematics is the solving of problems. But the pulsations of this lively and active "heart of mathematics" had become almost invisible in the Greek heritage. What was left were the polished patterns on the skin. Descartes, and other mathematicians of the seventeenth century, suspected that the Greeks had been in possession of a powerful method, an *ars analytica*, that had made it possible for them to arrive at their results. He even accused them of deliberately hiding it. This suspicion partly found its origin in a description of the method of analysis by Pappus in his *Mathematical Collection*, that had been translated in 1589 by Commandino. The development of modern algebra actually started as an attempt at restoring the lost *ars analytica*.[114] The suspicion that the Greeks were "cheating" has been further vindicated by the discovery in 1906 by Heiberg of an unknown work of Archimedes: *The Method*.[115] In this work Archimedes reveals what heuristics had led him to the discovery of some of his theorems. In his "secret reasoning" he seems to have used strong mechanical images, that were taboo in geometry.

112. See H.J.M. Bos, "Arguments on Motivation in the Rise and Decline of a Mathematical Theory; the 'Construction of Equations', 1637-*ca.*1750", in *Archive for History of Exact Sciences*, XXX (1984), p. 364.

113. B.L. Van der Waerden, *op. cit.*, p. 266.

114. M.S. Mahoney, *The Mathematical Career of Pierre de Fermat*, pp. 28ff.

115. T.L. Heath ed., *The Method of Archimedes. Recently Discovered by Heiberg. A Supplement to the Works of Archimedes*, Cambridge, 1912.

One thing is clear: from the sixteenth century onwards, the Greek heritage no longer satisfied the needs of a mathematics increasingly oriented towards the solving of problems. The tension caused by the prohibition of numbers from the realm of geometrical magnitude,[116] creating an obstacle to applied mathematics, is manifest beneath the surface of the Chinese Euclid, hidden in the commentary. In his preface, Ricci had promised that mathematics would be of great help for almost any aspect of life. But with the naked text of Euclid it is not at all obvious how to proceed and use geometry. Even the truly fundamental Pythagorean theorem only establishes a relation between three squares and doesn't tell you how it can be of immeasurable value in calculating heights and distances.

There are many styles of doing mathematics. Chinese mathematics was not very different in style from much that was going on in Europe at the time of the translation of Euclid, such as the algebraic tradition taken over from the Arabs, a tradition reaching back to the Babylonians. According to Mahoney, "throughout the sixteenth and early seventeenth centuries, mathematics meant many different things to many different people".[117] Euclid was only a part of what went into the making of modern mathematics, but it was by far the most visible part because it was pursued within an academic setting. Most mathematics was pursued outside the walls of the university.

The question might even be raised whether Euclid was truly fundamental to the rise of modern science. In the eyes of Bruins, a modern physicist-mathematician and historian of mathematics, the methods of the Babylonians remain valuable, whereas of the Euclidian "demonstrations" nothing is left.[118] From a modern point of view Euclid's work is a failure and he did not succeed in his purpose of building a flawless foundation for mathematics.[119] He tried to make mathematics independent of the senses, but in fact he only drew conclusions from figures.[120] Of course it falls far outside the scope of this paper and of my capabilities to try to evaluate the importance of Euclid for the creation of the modern world. I only wanted to give voice to some alternative points of view that question an all too easy equation of Euclid and Western scientific thinking and success.

In summary, I have tried to show in this paper, how, in comparing aspects of Western and Chinese culture, we should be very careful about our points of departure. In the case of the translation of Euclid, we have, in a literal way, in the first place to establish what text exactly was transmitted. I have indicated some noteworthy differences between the transmitted text and the original. These differences were seen to result partly from the very complicated textual transmission of Euclid, but they also point to an interpretation and

116. Cf. one of Murdoch's concluding remarks about Medieval proportion-theory: "The final, and I think in many ways most important, distinguishing mark which the medieval history of proportions exhibits is its consistent tendency to read arithmetical conceptions into the geometrical, and into theories dealing with general magnitude. In effect, number was being considered an element of geometry; the Greek distinction between the continuous and the discrete was beginning to undergo erosion."

117. M.S. Mahoney, *op. cit.*, p. 2.

118. E.M. Bruins, "Babylone et Héron versus Euclide", *Revue d'Assyriologie et d'archéologie orientale*, 58 (1964), pp. 173-181, p. 181: "Ce n'est que dans les dernières décades que l'on s'est rendu compte du fait que les méthodes des Babyloniens, les méthodes de Héron restent valables, tandis que, de la 'démonstration d'Euclide', il ne reste rien."

119. For an analysis of some of the flaws in Euclid, see A. Seidenberg, "Did Euclid's Elements, Book I, Develop Geometry Axiomatically?", in *Archive for History of Exact Sciences*, XIV (1974/75), pp. 263-295.

120. E.M. Bruins, *op. cit.*: "Euclide n'a pas fait autre chose que tirer ses conclusions des figures qu'il a dessinées."

understanding of Euclid that was quite different from the modern point of view. Although it is difficult to get a general picture of the European understanding of Euclid — a lot of work remains to be done —, there are enough indications that the difference in interpretation is far greater than is generally recognized. The text of the *Elements* found its way to Western Europe without the explanations of modern scholarship and caused serious problems of understanding. These problems become manifest particularly in the commentary and in the translation of the key-words of the axiomatic method, words that, through their diachronic constancy, have masked the subtle changes in meaning of the underlying concepts. Probably the most remarkable divergence from the Greek Euclid is the intrusion of arithmetical concepts into pure geometry. The over-all result is, that it is almost impossible to expect that the text would readily have been understood.

In a broader sense, by placing the Chinese Euclid in its European context, I have tried to draw attention to the difficulties of comparing Western and Chinese thought. On the face of it, Euclid is a major point of difference between Chinese and Western thought, and dichotomies such as abstract versus concrete, general versus particular, theoretical and speculative versus practical etc. easily impose themselves. But I believe that it is important to realise, before we study the reception of Euclid in China, that Euclid was in some ways just as alien to Europe as to China.

GLOSSARY

gonglun	公論
jie xiang zhi shu	借象之術
Jihe yuanben	幾何原本
Li Zhizao	李之藻
lun	論
mingshu	命數
san (si) chengfang	三(四)乘方
shujia	數家
Shuli jingyun	數理精蘊
Tianxue chuhan	天學初函
Tianzhu shiyi	天主實義
xiangjie	相結
xiao he	小合
Xiguo jifa	西國記法
Xu Guangqi	徐光啓
Yuanrong jiaoyi	圜容較義
Zhongguo shixue congshu	中國史學叢書
zhonglü	中率